

B.Sc/6th Sem (G)/MTM/25(CBCS)

2025

6th Semester Examination
MATHEMATICS (General)

Paper : DSE 1B/2B/3B-T

[CBCS]

Full Marks : 60

Time : Three Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers

in their own words as far as practicable.

[Mechanics]

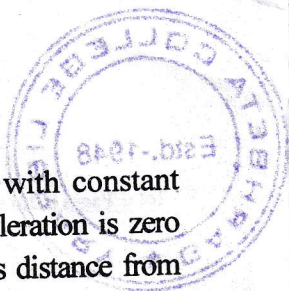
Group - A

Answer any *ten* questions with brief explanations :

$2 \times 10 = 20$

1. If the tangential and normal accelerations of a particle moving in a plane curve are equal then find an expression for its velocity.
2. State Newton's laws of motion.
3. A particle is moving with a constant velocity parallel to the axis of y and a velocity proportional to y parallel to the axis of x . Show that it will describe a parabola.

P.T.O.



4. A particle describes the curve $r = ae^{\theta}$ with constant angular velocity. Show that the radial acceleration is zero and the transverse acceleration varies as its distance from the pole.
5. If P be the power of a force F and v the velocity of its point of application then prove that $P = Fv$.
6. What is meant by potential energy? Find the potential energy of a particle of mass m situated at a height h from earth's surface.
7. A particle moves along a straight line according to the law $s^2 = 6t^2 + 4t + 3$, where s is the distance at time t . Find the velocity of the particle at time $t = 1$.
8. Two unequal masses m_1 and m_2 are connected by a light inextensible string over a smooth fixed pulley. Find the tension of the string.
9. Suppose a 5 kg weight tied to a string swings back and forth 5 times in 20 seconds. How long is the string?
10. If a particle moves in a straight line with uniform acceleration, prove that increase (or change) in K.E. is equal to the work done by the acting force.
11. If a bi-cyclist always works $\frac{1}{10}$ H.P. and goes 12 miles per hour on the level ground, find the resistance of the road.
12. The velocity of a point parallel to the axes x and y are $u + wy$ and $v + wx$ respectively when u, v, w are constants. Show that the path of the point is a conic.

13. If a particle moves with constant angular velocity about a point O in its plane of motion, show that its cross-radial acceleration is proportional to its radial velocity, O being the origin.
14. If the velocity of a particle at a distance x from a fixed point O is given by $v^2 = a - bx^2$, where a, b are constants, prove that the motion is simple harmonic.
15. Write the necessary and sufficient conditions of equilibrium for the system of co-planar forces acting on a rigid body.

Group - B

Answer any *four* questions : 5×4=20

16. A particle of unit mass is projected with velocity u at an inclination α above the horizon in a medium whose resistance is k -times the velocity. Show that the direction of the path described will again make an angle α with the horizon after a time $\frac{1}{k} \log \left(1 + \frac{2ku}{g} \sin \alpha \right)$.
17. If a hemisphere rests in equilibrium with its curved surface in contact with a rough plane inclined to a horizontal at an angle θ then show that the inclination of the plane of the hemisphere to the horizontal is $\sin^{-1} \left(\frac{8}{3} \sin \theta \right)$, provided $\theta < \sin^{-1} \frac{3}{8}$.

P.T.O.

18. The resultant of two like parallel forces P, Q passes through a point C . When P is increased by R and Q by S the resultant still passes through S and also when Q, R replace P, Q respectively. Prove that

$$S = R - \frac{(Q-R)^2}{P-Q}.$$

19. A square hole is punched out of a circular lamina, the diagonal of the square being a radius of the circle. Show

that the C.G. of the remainder is at a distance $\frac{a}{8\pi - 4}$

from the centre of circle of radius $\frac{a}{2}$.

20. A particle is performing a SHM of period T about a centre O and it passes through a point P where $OP = b$ with velocity v in the direction OP . Prove that the time

which elapses before it return to P is $\frac{T}{\pi} \tan^{-1} \left(\frac{vT}{2\pi b} \right)$.

21. An engine is pulling a train and works at constant power doing H unit of work per second. If M be the mass of the train and F be the resistance supposed to be constant, show that the time of generating the velocity v from rest is

$$\left[\frac{MH}{F^2} \log \frac{H}{(H - Fv)} - \frac{Mv}{F} \right] \text{ second.}$$

24. (i) A uniform cubical box of edge a is placed on the top of a fixed sphere. Show that the least radius of the sphere for which equilibrium will be stable is $\frac{a}{2}$.
- (ii) Find the distance from the mean position where the kinetic energy in a simple harmonic oscillator is equal to potential energy. 5+5
25. (i) Find the co-ordinates of the C.G. of a lamina in the shape of a quadrant of the curve $x^{2/3} + y^{2/3} = a^{2/3}$, density at any point (x, y) is given by $\rho = kxy$, (k being constant).
- (ii) Find the C.G. of the surface of revolution of the curve $y = f(x)$ about x axis. 7+3
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OR

[Linear Programming]

Group - A

Answer any *ten* from the following : $2 \times 10 = 20$

1. Define extreme point. Find the extreme point (if any) of the set $S = \{(x, y) | x^2 + y^2 \leq 25\}$.
2. Define slack and surplus variable.
3. Show that the vectors $(2, 2, 8)$, $(1, 0, 4)$ and $(1, 2, 4)$ are linearly dependent.
4. Find a basic solution for the equations :
 $x_1 + 3x_2 + 2x_3 + 3x_4 = 10$; $2x_1 - x_2 + 4x_3 + 6x_4 = 16$.
5. Define convex hull and convex polyhedron.
6. Show that the set of points on the line $y = mx + c$ is convex.
7. Put the following problem in a maximization form :

$$\text{Minimize } Z = 3x_1 - x_2 - x_3$$

$$\text{Subject to } x_1 + 3x_2 - 4x_3 \leq 12;$$

$$x_1 - x_2 + x_3 \leq 20;$$

$$x_1 - 4x_2 - 5x_3 \geq 5,$$

$x_1 \geq 0$, x_2 and x_3 are unrestricted in sign.

P.T.O.

8. Find the dual of the following L.P.P. :

$$\text{Maximize } Z = 2x_1 + 3x_2 + x_3$$

$$\text{Subject to } 4x_1 + 3x_2 + x_3 = 6;$$

$$x_1 + 2x_2 + 5x_3 = 4;$$

$$x_1 + 4x_2 \leq 420,$$

$$x_1, x_2, x_3 \geq 0.$$

9. Show that the intersection of two convex set is also a convex set.
10. Define feasible solution and basic feasible solution.
11. State the fundamental theorem of L.P.P.
12. What are the limitations of graphical method to solve a L.P.P.?
13. What do you mean by cycling in L.P.P.? How it can be resolved?
14. Find a basis of E^3 containing the vectors $(1, 1, 2)$ and $(3, 5, 2)$.
15. Explain the degenerate and non-degenerate solution of an L.P.P.

Group - B

Answer any **four** from the following : $5 \times 4 = 20$

16. Prove that $x_1 = 2$, $x_2 = 1$, $x_3 = 3$ is the feasible solution of the set of equations

$$4x_1 + 2x_2 - 3x_3 = 1; -6x_1 - 4x_2 + 5x_3 = -1;$$

Reduce the solution to a basic feasible solution.

17. Use graphical method to solve the following L.P.P. :

$$\text{Minimize } Z = 20x_1 + 10x_2$$

$$\text{Subject to } x_1 + 2x_2 \leq 40;$$

$$3x_1 + x_2 \geq 30;$$

$$4x_1 + 3x_2 \geq 60$$

$$x_1, x_2 \geq 0.$$

18. Prove that a basic feasible solution to a linear programming problem corresponds to an extreme of the convex set of feasible solutions.
19. A furniture manufacturer wishes to determine the number of tables and chairs to be made by him in order to optimize the use of his available resources. These products utilize two different types of timber and he has on hand 1500 board feet of the first type and 1000 board feet of the second type. He has 800 man-hours available for the total job. Each table and chair requires

P.T.O.

5 and 1 board feet respectively, of the first type of timber and 2 and 3 board feet respectively of the second type 3 man-hours are required to make a table and 2 man-hours are needed to make a chair. The manufacturer makes a profit of Rs. 12.00 on a table and Rs. 5.00 on a chair. Write out the complete linear programming formulation of the problem in terms of maximizing the profit.

20. If any constraints in the primal problem be a perfect equality then show that the corresponding dual variable is unrestricted in sign.
21. Solve the following L.P.P. using simplex table :

$$\text{Maximize } Z = x_1 + x_2 + 3x_3$$

$$\text{Subject to } 3x_1 + 2x_2 + x_3 \leq 3;$$

$$2x_1 + x_2 + 2x_3 \leq 2;$$

$$x_1, x_2, x_3 \geq 0.$$

Group - C

Answer any *two* from the following : $10 \times 2 = 20$

22. Find all basic feasible solutions of the following set of equations :

$$2x_1 + 6x_2 + 2x_3 + x_4 = 3;$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2;$$

$$x_1, x_2, x_3, x_4 \geq 0$$

23. Use Charne's Big-M method to solve the following L.P.P. :

$$\text{Maximize } Z = 5x_1 - 2x_2 + 3x_3$$

$$\text{Subject to } 2x_1 + 2x_2 - x_3 \geq 2;$$

$$3x_1 - 4x_2 \leq 3;$$

$$x_2 + 3x_3 \leq 5;$$

$$x_1, x_2, x_3 \geq 0.$$

24. Solve the following LPP using two phase method :

$$\text{Minimize } Z = 4x_1 + x_2$$

$$\text{Subject to } 3x_1 + x_2 = 3;$$

$$4x_1 + 3x_3 \geq 6;$$

$$x_1 + 2x_2 \leq 4;$$

$$x_1, x_2 \geq 0.$$

25. If X be any feasible solution to the primal problem and V be any feasible solution to dual problem then show that $CX \leq b^T V$.

P.T.O.

OR

[Numerical Methods]**Group - A**Answer any *ten* questions : $2 \times 10 = 20$

1. Discuss the rate of convergence of Secant method.
2. What is interpolation?
3. Define single step method. Give example.
4. What is pivoting? Why pivoting is important?
5. Prove that $Dy = \frac{1}{h} \left[\Delta y - \frac{\Delta^2 y}{2} + \frac{\Delta^3 y}{3} \dots \dots \right]$ where $e^{hD} = 1 + \Delta$.
6. When does the Newton-Raphson method fails?
7. What are the advantages of Lagrange's interpolation formula?
8. State the condition of Convergence of N-R method.
9. Find the number of significant figures in $V_A = 1.8921$, given its relative error as 0.1×10^{-2} .
10. Prove the relation $\Delta^n = (E - 1)^n$.
11. Explain the geometrical interpretation of N-R method.
12. Write the convergence criteria of the Gauss-Jacobi method to solve a system of linear equation.

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13. If $f(x)$ be a polynomial of degree 2, prove that

$$\int_0^1 f(x) dx = \frac{1}{12} [5f(0) + 8f(1) - f(2)].$$

14. Using the table

$x:$	1	3	4	6
$f(x):$	-3	9	30	132

compute $f(2)$ by Newton's divided difference formula.

15. Write the computational formulae for solving $y' = f(x, y)$ with $y(x_0) = y_0$ by second order Runge-Kutta method. What is the advantages of this method w.r.t. Taylor's series method?

Group - B

Answer any *four* questions : $5 \times 4 = 20$

16. Establish Newton's Forward interpolation formula taking equally spaced interpolating points.
17. Use Euler's method with $h = 0.1$ to find the solution of $\frac{dy}{dx} = x + y^2$, $y(0) = 0$ at $x = 0.5$.
18. Describe Gauss-Jacobi's method for solving a system of linear equation.
19. Use method of iteration to find the root of the equation $\cos x - 3x + 1 = 0$ correct to 4 decimal places.

P.T.O.

20. Evaluate $\int_0^{\pi/2} \sqrt{\sin x} dx$ taking $n = 6$, correct upto four significant figures by

(i) Simpson's one-third rule

(ii) Trapezoidal rule.

21. Find the first, second and third derivative of the function $f(x)$ tabulated below at the points $x = 1.5$ and $x = 4.0$.

$x :$	1.5	2.0	2.5	3.0	3.5	4.0
$f(x) :$	3.375	7.000	13.625	24.000	38.875	59.000

Group - C

Answer any *two* questions : $10 \times 2 = 20$

22. Write down the formulas for Newton's forward and backward interpolation method. Mention the basic assumption on which these two formulas have been developed.

In a country school going children of a certain age group is given for different years as follows :

Year :	1950	1955	1960	1965	1970
No. of students : (in thousand)	304	329	357	387	421

Estimate the number in the year 1975.

23. (i) Estimate the missing term in the following table :

$x:$	0	1	2	3	4	5
$f(x):$	0	?	8	15	?	35

- (ii) Use Gauss-Seidal iteration method to solve the following system of equation.

$$7x_1 + 2x_2 - x_3 = 17.20$$

$$-x_1 + 9x_2 + 2x_3 = 18.90$$

$$x_1 + 5x_2 - 11x_3 = 28.50$$

4+6

24. (i) Use the Runge-Kutta method to approximate y when $x = 0.1$ and $x = 0.2$ given that $y = 1$ when

$$x = 0 \text{ and } \frac{dy}{dx} = x + y.$$

- (ii) Find a real root of the equation $x^3 + 2x - 2 = 0$ by Regula-Falsi method. Correct upto four significant figures.

5+5

25. (i) Estimate the absolute error and relative error for the function $f(x) = \sqrt{x} + x$ for $x_a = 4.000$.

- (ii) Show that $D = h \left(1 + \frac{1}{2} \nabla + \frac{5}{12} \nabla^2 + \dots \right) Dy_n$

where D is the differential operator.

- (iii) Define the forward difference operator Δ . For any positive integer n show that

$$\Delta^n y_0 = \sum_{i=0}^n (-1)^i \binom{n}{i} y_{n-i}.$$

3+3+4

P.T.O.

OR

[Integer Programming and Theory of Games]**Group - A**Answer any *ten* questions : $2 \times 10 = 20$

1. State the fundamental theorem of rectangular game.
2. Define the payoff function and saddle point.
3. For what value of a , the game with the following payoff matrix is strictly determinable?

		B		
		I	II	III
A	I	a	5	2
	II	-1	a	-8
	III	-2	3	a

4. What are the traditional approaches to solving integer programming problem?
5. What is the principle behind the Gromory Cut?
6. Define a two-person zero sum or a rectangular game.
7. What are pure and mixed strategies?
8. Two players A and B match coins. If the coins match, then A wins one unit of the value and if the coins do not match, then B wins one unit of the value. Determine the optimum strategies for the plays and the value of the game.

9. Write a short note on application of game theory.
10. What is mixed integer programming problem?
11. Write the basic characteristic of game theory.
12. Write the dominance property of a game.
13. What is the need of integer programming?
14. How does the optimal solution of an IPP compare with that of the LPP?
15. Write two method names for solving mixed IPP.

Group - B

Answer any **four** questions : $5 \times 4 = 20$

16. The payoff matrix of a two-person zero-sum game is

		B		
		B_1	B_2	B_3
A	A_1	1	2	1
	A_2	0	-4	-1
	A_3	1	3	-2

Determine the number of saddle points and the corresponding optimal solutions. $2+3$

P.T.O.

17. Use the dominance to reduce the payoff matrix and solve the game with the following payoff matrix :

		B		
		6	8	6
A	4	6	12	2
	2	4	12	2

18. Prove that if p^* and q^* be the optimal strategies then $E(p^*, q) \geq v \geq E(p, q^*)$, where v is the value of the game.
19. Players A and B play a game in which each has three coins a 50 paise, a Re. 1 and a Rs. 2. Each player selects a coin without the knowledge of the other's choice. If the sum of the coins is an odd amount, A wins B's coin; if the sum is even, B wins A's coin. Find the best strategy for each player and the value of the game.
20. Derive the Gomory's constraint for general IPP?
21. Formulate the following game as a linear programming problem.

		B		
		1	2	3
A	1	4	1	-3
	2	3	1	6
	3	-3	4	-2

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Group - C

Answer any *two* questions : $10 \times 2 = 20$

22. Use Gromory's cutting plane method to find the optimal solution of the L.P.P.

$$\text{Maximize } Z = 4x_1 + 3x_2$$

$$\text{Subject to } 3x_1 + 4x_2 \leq 12,$$

$$4x_1 + 2x_2 \leq 9,$$

$$x_1, x_2 \geq 0 \text{ and all are integers.}$$

23. Reduce the following game to $m \times 2$ game and solve by graphical method.

		Player B			
		y_1	y_2	y_3	y_4
Player A	x_1	19	6	7	5
	x_2	7	3	14	6
	x_3	12	8	18	4
	x_4	8	7	13	-1

24. Write both primal and dual linear programming problems corresponding to the following rectangular game and solve one of the LPP.

		Y			
		y_1	y_2	y_3	y_4
X	x_1	5	7	4	10
	x_2	4	3	7	9
	x_3	1	2	5	6

P.T.O.

(20)

25. Solve the following IPP using branch-and-bound method :

$$\text{Maximize } Z = 2x_1 + 2x_2$$

$$\text{Subject to } 5x_1 + 3x_2 \leq 8$$

$$x_1 + 2x_2 \leq 4$$

and $x_1, x_2 \geq 0$ are integers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50