



B.Sc./6th Sem (H)/MTM/25(CBCS)

2025

6th Semester Examination
MATHEMATICS (Honours)

Paper : DSE 4-T

[CBCS]

Full Marks : 60

Time : Three Hours

*The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.*

[Mathematical Modelling]

Group - A

Answer any **ten** questions : $2 \times 10 = 20$

1. Evaluate $L \left\{ \int_0^t \frac{\sin u}{u} du \right\}$ by the help of initial value theorem.
2. What is Monte Carlo simulation?
3. Find $L^{-1} \left\{ \frac{3s+7}{s^2-2s-3} \right\}$.
4. Define exponential order on Laplace Transformation.
Show that the function t^2 is of exponential order 3.

P.T.O.

5. Determine the singular points of the differential equation

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 6y = 0 \text{ and classify as regular or irregular.}$$

6. What do you mean by sensitivity analysis in LPP?

7. Show that although $(2, 3, 2)$ is a feasible solution to the system of equations $x_1 + x_2 + 2x_3 = 9$, $3x_1 + 2x_2 + 5x_3 = 22$, $x_1, x_2, x_3 \geq 0$. It is not a basic solution.

8. Write the limitations of the simulation technique.

9. Verify the initial value theorem in connection with Laplace transform for the function $2 + \cos(t)$.

10. What is probabilistic process?

11. Using first shifting theorem find $L^{-1} \left\{ \frac{s}{s^2 + 2s + 5} \right\}$.

12. Write the step of simulation process.

13. Find $L \{t \sin 4t\}$.

14. Find the Laplace transformation of $t \frac{d^2 y}{dt^2} + \frac{dy}{dt} + ty = 0$,
 $y(0) = 1, y'(0) = 0$.

15. If $L[f(t)] = \frac{k}{s(s^2 + 4)}$ and $\lim_{t \rightarrow \infty} f(t) = 1$, then find the value of k .

(3)

Group - B

Answer any **four** questions : 5×4=20

16. Consider an LPP

Maximize $z = cX$

subject to $AX = b, X \geq 0,$

and suppose an optimal basic feasible solution has been found. Now let the right-hand-side value b_i of a constraint be changed by an amount Δb_i . If this constraint correspond to a basic variable in the current solution, determine the range of Δb_i for which the current basis remains optimal and feasible.

17. Find $L^{-1} \left\{ \frac{s-1}{(s+3)(s^2+2s+1)} \right\}$.

18. Write an algorithm to calculate the volume trapped between cone $z = \sqrt{x^2 + y^2}$ and hemisphere $z = \sqrt{1 - x^2 - y^2}$ using Monte Carlo simulation.

19. Let $f(t)$ and its derivatives $f'(t), f''(t), \dots, f^{n-1}(t)$ be continuous for all $t \geq 0$ and be of exponential order $e^{\alpha t}$ and if $f^n(t)$ is piecewise continuous in every closed

P.T.O.

(4)

interval $0 \leq t \leq b$ ($b > 0$), then prove that $L\{f^n(t)\}$ exists when $s > \alpha$ and given by

$$L\{f^n(t)\} = s^n L\{f(t)\} - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) - \dots - f^{n-1}(0).$$

20. Prove that $[J_{1/2}(x)]^2 + [J_{-1/2}(x)]^2 = \frac{2}{\pi x}$ where

$J_n(x)$ denotes Bessel function of order n .

21. Using Laplace transform evaluate $\int_0^\infty \frac{x \sin xt}{x^2 + a^2} dx$,
 $a, t > 0$.

Group - C

Answer any **two** questions : $10 \times 2 = 20$

22. (i) Solve the Legendre differential equation of the form

$$(1-z^2) \frac{d^2 y}{dz^2} - 2z \frac{dy}{dz} + 20y = 0.$$

(ii) Express a polynomial $3x^3 + 5x^2 - 2x + 9$ in terms of Legendre polynomials. $8+2$

23. Find the optimal solution of the LPP

$$\text{Maximize } z = 4x_1 + 3x_2$$

$$\text{subject to } x_1 + x_2 \leq 5,$$

$$3x_1 + x_2 \leq 7,$$

$$x_1 + 2x_2 \leq 10,$$

$$x_1, x_2 \geq 0$$

Show how to find the optimal solution of the problem if

- (i) The first component of the original requirement vector be increased by one unit and third component be decreased by one unit.
- (ii) The second component of the requirement vector be decreased by two units. 5+5

24. (i) Apply convolution theorem to evaluate

$$L^{-1} \left\{ \frac{s^2}{(s^2 + a^2)^2} \right\}.$$

$$(ii) \text{ Solve } (tD^2 + (1-2t)D - 2)y = 0, \quad y(0) = 1,$$

$$y'(0) = 2, \text{ where } D \equiv \frac{d}{dt}. \quad 5+5$$

P.T.O.

25. (i) Write a Monte Carlo simulation algorithm for a harbour with unloading facilities for ships finding the answers of the following questions :
- (a) What is the average and maximum times per ship in the harbour?
 - (b) What are the average and maximum waiting times per ship?
 - (c) What percentage of the time are the unloading facilities idle?
- (ii) What are the disadvantages of linear congruence method to generate random numbers? 7+3
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OR

[Differential Geometry]**Group - A**Answer any *ten* questions : $2 \times 10 = 20$

1. Define Frenet frame.
2. Define a space curve. Give an example of a space curve and describe its motion.
3. What is torsion? How is torsion related to the curvature of a space curve?
4. What is Rodrigues' formula in differential geometry?
5. What are the first and second fundamental forms of a surface?
6. State Euler's theorem for a surface.
7. Define the direction coefficients of a surface.
8. What are conjugate and asymptotic lines on a surface?
9. Define the torsion of a geodesic, and explain its significance.
10. Define a minimal surface, and give an example of a minimal surface.
11. Compute the curvature of the curve γ given by

$$\gamma(t) = \left(\frac{4}{5} \cos t, 1 - \sin t, -\frac{3}{5} \cos t \right).$$

P.T.O.

12. Prove that the curve γ given by $\gamma(t) = (a \cos t, b \sin t)$ is a convex curve for two real constants a and b .
13. Show that a curve on a surface is a geodesic if and only if its geodesic curvature is zero everywhere.
14. Is Gaussian curvature of a surface invariant? Justify.
15. Find a curve which is the intersection of two surfaces, $y = x^2$ and $xz = y^2$.

Group - B

Answer any *four* questions : 5×4=20

16. Prove that for a helix the ratio of curvature and torsion is constant.
17. State the Gauss-Bonnet theorem, and explain its significance in differential geometry.
18. Give an example of a surface that has constant mean curvature, and explain its geometric properties.
19. Prove that the curve obtained by the intersection of the cylinders $F : y = x^2$ and $G : z = x^3$ is regular. Hence obtain its parametric equation.
20. Define the Weingarten map of a surface, and explain how it relates to the first and second fundamental forms.

21. Prove that the surface $\sigma(t, \theta) = \left(\left(1 - t \sin \frac{\theta}{2} \right) \cos \theta, \left(1 - t \sin \frac{\theta}{2} \right) \sin \theta, t \cos \frac{\theta}{2} \right); \frac{-1}{2} < t < \frac{1}{2}, 0 < \theta < 2\pi$ is not orientable.

Group - C

Answer any **two** questions : $10 \times 2 = 20$

22. (i) Find the area on a surface $\vec{r} = (u \cos v, u \sin v, u)$ within the patch $u = 1$ to $u = 3$ and $v = 0$ to $\frac{\pi}{4}$.
- (ii) If k is curvature of a curve on a surface whose normal curvature and the geodesic curvatures are k_n and k_g then prove that $k^2 = k_n^2 + k_g^2$. $5+5$
23. (i) Compute the second fundamental form of the elliptic paraboloid $r(u, v) = (u, v, u^2 + v^2)$.
- (ii) Show that $z = f(x, y)$ where f is a smooth function of two variables, is a minimal surface if and only if $(1 + f_x^2)f_{yy} - 2f_x f_y f_{xy} + (1 + f_y^2)f_{xx} = 0$. $5+5$

P.T.O.

24. (i) State Clairaut's theorem for geodesics, and explain its geometric significance.
- (ii) State the Bonnet-Myers theorem, and explain its significance in the study of geodesics. 5+5
25. (i) Find the differential equation of the geodesic for the metric $ds^2 = (du)^2 + (\sin u)^2 (dv)^2$.
- (ii) Show that the geodesic curvature of the curve $u = c$ with the metric $\lambda^2 (du)^2 + \mu^2 (dv)^2$ is
$$\frac{1}{\lambda \mu} \frac{\partial \mu}{\partial u}.$$
 5+5
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OR

[Bio Mathematics]**Group - A**Answer any **ten** questions :

2×10=20

1. What is the growth limiting factors for microbial growth in a Chemostat?
2. Define the terms immigration and emigration.
3. What are the physical significance of the dominant eigen value of the Leslie matrix?
4. Write short note on Routh-Hurwitz criterion.
5. Discuss the Allee effect given by

$$\frac{dN}{dt} = N \left[k_0 - l(N - \mu)^2 \right], \quad \mu < \sqrt{\frac{k_0}{l}}, \text{ where } k_0 \text{ and } \mu$$
are positive constants.
6. Define Lyapunov stable and uniformly stable.
7. During an epidemic, an infected person can be removed from the scene of disease by quarantine and a susceptible person can be made immune by vaccination. If vaccination is performed at a rate α , then formulate the model by differential equations with initial conditions.
8. Formulate the Kermack-McKendrick SEAIR model based on the epidemic model of Influenza. Where,

P.T.O.

E represents the people going through the transition stage from infected to infectious. In the class E , a significant number of persons never develop symptoms, but they are capable of transmitting the disease which is known as A class. S , I and R has the usual meaning in epidemic model.

9. Define the term Centre, Stable node and Unstable node.
10. Find the eigen value of the Leslie matrix

$$A = \begin{bmatrix} 0 & 9 & 12 \\ \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \end{bmatrix} . \text{ Also, find the dominant eigen value,}$$

if exists.

11. Write down the Michaelis-Menten kinetics equation and explain each term.
12. Define the Holling Type II functional response.
13. Define the basic SI and SIR epidemic models.
14. State the one-species diffusion model (Fisher's equation).
15. What is the Nicholson-Bailey host-parasitoid model?

Group - B

Answer any *four* questions : $5 \times 4 = 20$

16. For a blood vessel of constant radius R , the length L and driving force $P = p_1 - p_2$, show that the average velocity of flow is equal to half the maximum velocity and the resistance is proportional to $\frac{L}{R^4}$.
17. In a population of birds, the proportionate birth rate and the proportionate death rate are both constant, being 0.45 per year and 0.65 per year respectively. Formulate a model of the population and discuss its long term behaviour.
18. Some diseases, such as typhoid fever are spread largely by carriers, individuals who can transmit the disease but who exhibit no over symptoms. Let x and y , respectively denote the proportion of susceptible are carriers in the population. Suppose that carrier are identified and removed from the population at a rate β , so that $\frac{dy}{dt} = -\beta y$. Suppose also that the disease spreads at a rate α , proportional to the product of x and y , thus $\frac{dx}{dt} = -\alpha xy$.
- (i) Determine the proportions of carriers at any time t , where $y(0) = y_0$.

P.T.O.

(ii) Find the susceptible at time t , where $x(0) = x_0$.

(iii) Find the proportion of the population that escapes the epidemic.

19. Show that for the system of equations

$$\frac{dx}{dt} = -y + x(x^2 + y^2 - 1) \quad \text{and} \quad \frac{dy}{dt} = x + y(x^2 + y^2 - 1)$$

is an unstable limit cycle, where $x = r \cos \theta$, $y = r \sin \theta$ and $r = 1$.

20. For the Lotka-Volterra system

$$\frac{dx}{dt} = \alpha x - \beta xy, \quad \frac{dy}{dt} = \delta xy - \gamma y,$$

sketch the phase plane, identify the critical points, and classify their type.

21. Show that Fisher's equation

$$\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2} + ru \left(1 - \frac{u}{K} \right)$$

admits a travelling wave solution $u(x, t) = U(z)$ with $z = x - ct$, and derive the minimal wave speed.

Group - C

Answer any *two* questions : 10×2=20

22. (i) Find out the steady state solution and discuss the stability at that of the prey-predator model

$$\frac{dN_1}{dt} = N_1(\alpha - \beta N_2); \alpha > 0, \beta > 0$$

$$\frac{dN_2}{dt} = -N_2(\gamma - \delta N_1); \gamma > 0, \delta > 0$$

where N_1 and N_2 represents the density of prey and density of predator respectively.

- (ii) Give the geometrical interpretation of the above prey-predator model.

23. The delayed Lotka-Volterra competition system is given by

$$\frac{dx(t)}{dt} = x(t)[2 - \alpha x(t) - \beta y(t-r)]; \alpha > 0, \beta > 0$$

$$\frac{dy(t)}{dt} = y(t)[2 - \gamma x(t-r) - \delta y(t)]; \gamma > 0, \delta > 0$$

- (i) Obtain the steady-state solutions (if exist).
- (ii) Investigate the stability of the non-zero steady-states for $\alpha = \delta = 2$ and $\beta = \gamma = 1$.

P.T.O.

24. Let $N(t)$ be the number of tiger population at any time t . The quotient of birth rate and death rate by the population size N are respectively by,

$$\frac{\text{Birthrate}}{N} = \frac{3}{2} + \frac{1}{1000}N \quad \text{and} \quad \frac{\text{Deathrate}}{N} = \frac{1}{2} + \frac{1}{3000}N.$$

Formulate a model (using differential equation) that describes the growth and regulation of this tiger population. Solve for $N(t)$, assuming $N(0) = 100$ and describe the long term behaviour of this tiger population as $t \rightarrow \infty$.

25. For the discrete host-parasitoid model

$$H_{n+1} = RH_n e^{-aP_n}, \quad P_{n+1} = cH_n (1 - e^{-aP_n}),$$

perform a linear stability analysis of the coexistence equilibrium and determine conditions for oscillatory dynamics.