



B.Sc./3rd Sem/MTM/25(NEP)

2025

3rd Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MJ 4-T (Single Core Major)

[Differential Equations and Vector Calculus]

Full Marks : 60

Time : Three Hours

The figures in the margin indicate full marks.

*Candidates are required to give their answers
in their own words as far as practicable.*

Group - A

Answer any *ten* questions : $2 \times 10 = 20$

1. Define fundamental set of solutions for system of ordinary differential equation.
2. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solution

of $x^2 \frac{dy}{dx^2} - 2x \frac{dy}{dx}$ for all x in $[0, 10]$ consider the

Wronskian $W(x) = y_1(x) \frac{dy_2}{dx} - \frac{dy_1}{dx} y_2(x)$. If $W(1) = 1$

then find the value of $W(3) - W(2)$.

P.T.O.

3. Show that $f(x, y) = xy^2$ on $\mathbb{R} : \{|x| \leq 1, |y| \leq 1\}$ satisfying a Lipschitz condition. But this function does not satisfy a Lipschitz condition on the strip $S : \{|x| \leq 1, |y| \leq \infty\}$.

4. If $y(x) = x^2 \sin x$ is a solution of an n -th order linear ODE

$$y''(x) + a_1 y^{n-1}(x) + \dots + a_{n-1} y'(x) + a_n y(x) = 0$$

with real constant coefficients, then prove that the least possible value of n is 6.

5. Evaluate $\int_0^1 \left(\vec{r} \times \frac{d^2 \vec{r}}{dt^2} \right) dt$, where $\vec{r} = t^3 \hat{i} + 2t^2 \hat{j} + 3t \hat{k}$.

6. If the volume of the tetrahedron is 2 cubic units and three of its vertices have position vectors $(1, 1, 0)$, $(1, 0, 1)$, $(2, -1, 1)$, find the locus of the fourth vertex.

7. Find the formula for curvature (κ) and torsion (τ) for the curve : $x = x(t)$, $y = y(t)$, $z = 0$.

8. Show that the point at infinity is a regular singular point of the differential equation $x^2 \frac{d^2 y}{dx^2} + (3x-1) \frac{dy}{dx} + y = 0$.

9. Given that $\vec{r}(t) = \begin{cases} 2\hat{i} - \hat{j} + 2\hat{k}, & \text{when } t = 2 \\ 4\hat{i} - 2\hat{j} + 3\hat{k}, & \text{when } t = 3 \end{cases}$. Show

$$\text{that } \int_2^3 \vec{r} \cdot \frac{d\vec{r}}{dt} dt = 10.$$

(3)

10. Given that $\vec{a} = 2\vec{l} - 3\vec{m} + 4\vec{n}$, $\vec{b} = \vec{l} + 2\vec{m} + 7\vec{n}$, $\vec{c} = 5\vec{l} - \vec{m} + 3\vec{n}$, where $\vec{l}$, $\vec{m}$, $\vec{n}$ are three non-coplanar vectors. Find $\vec{a} \cdot \vec{b} \times \vec{c}$.

11. Show that $y = x$ and $y = (x + 1)^{-1}$ are linearly independent solutions of

$$(2x+1)(x+1)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = (2x+1)^2.$$

12. Examine the nature and stability of the critical point of the system $\frac{dx}{dt} = -4x - y$, $\frac{dy}{dt} = x - 2y$.

13. Find the volume of the parallelopiped whose edges are represented by $(3\hat{i} + 2\hat{j} - 4\hat{k})$, $(3\hat{i} + \hat{j} + 3\hat{k})$ and $(\hat{i} - 2\hat{j} + \hat{k})$.

14. If $\vec{u} = t^2\hat{i} - t\hat{j} + (2t+1)\hat{k}$, $\vec{v} = (2t-3)\hat{i} + \hat{j} - t\hat{k}$ then find the value of $\frac{d}{dt}(\vec{u} \cdot \vec{v})$ at $t = 1$.

15. If $\vec{r} = \vec{a} \cos nt + \vec{b} \sin nt$ then show that

$$\frac{d^2\vec{r}}{dt^2} + n^2\vec{r} = \vec{0}.$$

P.T.O.

(4)

Group - B

Answer any *four* questions : $5 \times 4 = 20$

16. Show that the solution of the ODE $\frac{dx}{dt} = 2x + y$ and $\frac{dy}{dt} = 3x$ satisfy the relation $3x + y = Ke^{3t}$, where K is a real constant.

17. Find a vector δ , which is perpendicular to both $\alpha = 4\hat{i} + 5\hat{j} - \hat{k}$ and $\beta = \hat{i} - 4\hat{j} + 5\hat{k}$ and which satisfies the relation $\delta \cdot \gamma = 21$ where $\gamma = 3\hat{i} + \hat{j} - \hat{k}$.

18. If $\vec{r} = (2x^2y - x^4)\hat{i} + (e^{xy} - y \sin x)\hat{j} + (x^2 \cos y)\hat{k}$,
then find $\frac{\partial^2 \vec{r}}{\partial x^2} \times \frac{\partial^2 \vec{r}}{\partial y^2}$.

19. Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$ by the method of undetermined coefficients.

20. Show that the Frenet-Serret formulae can be written in the form :

$$\frac{d\hat{t}}{ds} = \vec{w} \times \hat{t}, \quad \frac{d\hat{n}}{ds} = \vec{w} \times \hat{n}, \quad \frac{d\hat{b}}{ds} = \vec{w} \times \hat{b}; \text{ determine } \vec{w}.$$

21. Find the values of ρ for which all solutions of

$$x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - \rho y = 0 \text{ tend to zero as } x \rightarrow \infty.$$

(5)

Group - C

Answer any *two* questions : $10 \times 2 = 20$

22. (a) The functions $y_1, y_2, \dots, y_n$ will be n linearly independent solutions of the equation

$$(D^n + P_1 D^{n-1} + P_2 D^{n-2} + \dots + P_n) y = F(x)$$

on $[a, b]$ if and only if $W(y_1, y_2, \dots, y_n)(x) \neq 0$,

$$\forall x \in [a, b].$$

- (b) Find the vector equation of the plane through the point $8\hat{i} + 2\hat{j} - 3\hat{k}$ and perpendicular to each

of the planes $\vec{r} \cdot (2\hat{i} - 5\hat{j} + 2\hat{k}) = 0$ and

$$\vec{r} \cdot (\hat{i} + 3\hat{j} - 5\hat{k}) + 5 = 0. \quad 5+5$$

23. (a) Let r and q be the roots of the indicial polynomial

for the equation $\frac{d^2 y}{dx^2} + a \frac{dy}{dx} + by = 0$, where a, b

are constants. If $r \neq q$, then show that two independent solutions are e^{rx} and e^{qx} .

- (b) Prove that

$$(\vec{b} \times \vec{c}) \cdot (\vec{a} \times \vec{d}) + (\vec{c} \times \vec{a}) \cdot (\vec{b} \times \vec{d}) + (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 0.$$

6+4

P.T.O.

24. (a) If $\vec{A} = x^2 yz \hat{i} - 2xz^3 \hat{j} + xz^2 \hat{k}$, $\vec{B} = 2z \hat{i} + y \hat{j} - x^2 \hat{k}$;

find $\frac{\partial^2}{\partial x \partial y} (\vec{A} \times \vec{B})$ at $(1, 0, -2)$.

- (b) Consider the system of linear differential equations :

$$\frac{dx}{dt} = -3x + y; \quad \frac{dy}{dt} = x - 3y.$$

Find the trajectories and phase curve in the phase plane of the system.

Discuss the stability of the system. 4+6

25. (a) Solve the differential equation

$$\frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} - \frac{1}{x^2} y = \log x$$

by the method of variation of parameters.

- (b) If $\vec{r} = (2x^2 y - x^4) \hat{i} + (e^{xy} - y \sin x) \hat{j} + (x^3 \cos y) \hat{k}$

then find $\frac{\partial^2 \vec{r}}{\partial x^2} \times \frac{\partial^2 \vec{r}}{\partial y^2}$.

5+5