

B.Sc./3rd Sem/PHS/25(NEP)

2025

3rd Semester Examination (CCFUP : NEP)

PHYSICS

Paper : MJ 3-T (Single Core Major)

(Waves and Optics with Mathematical Methods - I)

Full Marks : 60

Time : Three Hours

The figures in the margin indicate full marks.

*Candidates are required to give their answers
in their own words as far as practicable.*

Group - A

Answer any **ten** questions : $2 \times 10 = 20$

1. Determine the radius and interval of convergence of the

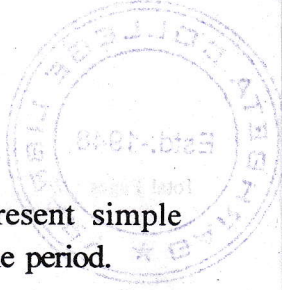
power series : $\sum_{n=0}^{\infty} \left(\frac{x}{3} \right)^n$.

2. A string of length L is fixed at both ends. It

satisfies $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$. Initially, $u(x, 0) = 0$ and

$\frac{\partial u}{\partial t}(x, 0) = V_0 \sin(3\pi x/L)$. Find $u(x, t)$.

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3. Does the function $\sin \omega t - \cos \omega t$ represent simple harmonic motion? If yes, calculate the time period.
 4. Explain why the central fringe in the Lloyd's mirror is black.
 5. Find the Fourier transform of $e^{a|t|}$, $a > 0$.
 6. Define temporal coherence.
 7. A reflecting telescope has an objective mirror of diameter of 1m. Find the minimum angular separation (in radians) between two stars when they are said to be just resolved using the telescope.
 8. Find the inverse Fourier transform of $F(\omega) = C$, C is a constant.
 9. In the Young's double slit experiment, the slit separation is 0.3 mm, the screen is 1.5 m away, and the fringe width is measured as 4.0 mm. Calculate the wavelength of the light used.
 10. Find $(f * g)(t)$ using the definition of convolution :
$$f(t) = \delta(t-1), \quad g(t) = \delta(t+2).$$
 11. Two SHM are superposed. What is the maximum possible amplitude and when does it occur?
 12. Write the intensity expression for a single slit diffraction and hence find the condition of minima of the pattern.

13. State the Leibnitz's conditions for alternating series.
14. How can you determine the refractive index of a given liquid through Newton's ring experiment? Assume the expression for n th ring as per your convenience.
15. How does the focal length of a Fresnel zone plate change if the wavelength of light used is doubled?

Group - B

Answer any **four** questions : $5 \times 4 = 20$

16. (a) Write the Laplace equation in cylindrical coordinates assuming no z -dependence.
 (b) Let us consider a circular cylinder of radius R with $V(R, \phi) = V_0 \sin 2\phi$ provided that $V(r=0)$ is finite. Solve the Laplace equation and find the potential inside it. 1+(3+1)
17. (a) Which of the following functions of x could be represented by a Fourier series over the range $[-\pi, \pi]$?
 (i) $x \sin(1/x)$
 (ii) $\tan x$
 (b) Determine whether the functions 1 and $\cos nx$ are orthogonal over $[-\pi, \pi]$. 3+2

P.T.O.

18. (a) Determine whether the following series are convergent :

$$(i) \sum_{n=1}^{\infty} \frac{[\ln(n)]^n}{n^{\frac{1}{2}}}$$

$$(ii) \sum_{n=1}^{\infty} \frac{n^{\frac{1}{2}}}{(n+1)^{\frac{1}{2}}}$$

- (b) What are the differences between the fringe systems in Newton's rings arrangement for transmitted light and reflected light? (2+2)+1

19. (a) A damped oscillator consists of a mass $m = 1$ kg, damping constant $b = 3$ kg/s, and spring constant $k = 40$ N/m.

- (i) Determine whether the system is underdamped, critically damped, or overdamped.

- (ii) Find the Quality Factor Q .

- (b) What is sharpness of resonance in forced oscillations? (2+2)+1

20. (a) A thin film of refractive index 1.50 is deposited on a glass substrate ($n = 1.80$). Light of wavelength 600.0 nm in air is normally incident. Determine the minimum film thickness for which the reflected light is destructive (first minimum).

- (b) State the results of the Stokes' treatment regarding the interference of light. 3+2

21. A long, thin rod of length L is initially at a uniform temperature T_0 . At time $t = 0$, both ends of the rod are suddenly brought into contact with ice blocks maintained at temperature 0°C . Assume the rod loses heat only through its ends and that heat conduction is related to the temperature distribution $u(x, t)$ along the rod with boundary conditions $u(0, t) = 0$ and $u(L, t) = 0$ for $t > 0$. The initial condition is $u(x, 0) = T_0$.

- (a) Solve the heat equation to determine the temperature $u(x, t)$ for all $t > 0$.

- (b) Explain physically why the higher Fourier modes decay faster than the lower ones. 4+1

Group - C

Answer any *two* questions : 10×2=20

22. (a) Compute the Fourier series of the periodic wave function defined by

$$f(x) = 1 \text{ for } 0 < x < \pi$$

$$= -1 \text{ for } -\pi < x < 0$$

- (b) Derive the equation of a Lissajous figure formed by the superposition of two perpendicular harmonic oscillations having frequencies in the ratio of 1 : 2

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and a phase difference of δ . Draw the corresponding Lissajous figures when $\delta = 0$ and π .

5+(3+2)

23. (a) What is meant by absolute and conditional convergence? Explain with examples.
- (b) Show that the Fourier transform of $f(t) = \cos(\omega_0 t)$ is equal to
- $$F(\omega) = \pi \left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0) \right].$$
- (c) Prove that for a spherically symmetric scalar function $f(r)$, the Laplacian in three dimensions reduces to $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r} \frac{df}{dr}$. Find all functions $f(r)$ that satisfy the radial Laplace equation $\nabla^2 f(r) = 0$. 3+3+(2+2)
24. (a) Obtain the intensity distribution for Fraunhofer diffraction pattern of a double slit.
- (b) How does the pattern of a double slit differ from that of a single slit?
- (c) Deduce the conditions for maxima and minima.
- (d) How the missing orders are obtained? 4+2+2+2
25. A string of a length l is fixed at the two ends and is plucked at a distance ' a ' from one end through a

(7)

distance h . Find the expressions for the displacement and velocity at any point x on the string at time t . State Young's law and show that the vibration of plucked string obeys the law. What modes of vibrations will be absent if the string is touched (i) at the plucked point and (ii) at one of the nodal points of the s th allowed mode of vibrations?

10