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B.Sc./1st Sem/MTM/25(NEP)

2025

1st Semester Examination (CCFUP : NEP)

MATHEMATICS

Paper : MJ 1-T (Single Core Major)

(Calculus, Geometry and
Ordinary Differential Equation)

Full Marks : 60

Time : Three Hours

The figures in the margin indicate full marks.

Candidates are required to give their answers

in their own words as far as practicable.

Group - A

Answer any *ten* questions : $2 \times 10 = 20$

1. Find the center of a conic represented by a second degree equation.
2. Solve $p^3 - 4xyp + 8y^2 = 0$.
3. Find the envelop (if any) of the line $x \cos \alpha + y \sin \alpha = 4$.
4. Find the eccentricity and the vertex of the

$$\text{conic } r = 3 \sec^2 \frac{\theta}{2}.$$

P.T.O.

(2)

5. Sketch the graph of $y = -\log(-x)$.

6. $\lim_{x \rightarrow 0^+} (\sin x)^{2 \tan x}$.

7. Find the Asymptotes of the curve

$$2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0.$$

8. Discuss the nature of the conic represented by

$$3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0.$$

9. Define general solution and singular solution of a differential equation.

10. Find the values of the constant λ such that

$$(2xe^y + 3y^2) \frac{dy}{dx} + (3x^2 + \lambda e^y) = 0 \text{ is exact.}$$

11. Using Leibnitz theorem, find the value of the n^{th} derivative for $y = e^{m \sin^{-1} x}$ at $x = 0$.

12. Find the points of inflexion of the curve $y = (\log x)^3$.

13. Show that the equation

$$x^2 - 2y^2 + 3z^2 - 4xy + 5yz - 6zx + 8x - 19y - 2z - 20 = 0$$

represents a cone.

14. Find the general equation of a quadratic cone containing all three axes.

15. Find the entire area of the cardioid $r = a(1 - \cos \theta)$.

Group - B

Answer any *four* questions : 5×4=20

16. Prove that

$$J_{m,n} = \int_0^{\pi} 2 \cos^m x \sin^n x dx = \frac{1}{2^{m+1}} \left[2 + \frac{2^2}{2} + \frac{2^3}{3} + \frac{2^4}{4} + \dots + \frac{2^m}{m} \right].$$

$$17. \text{ Prove that } e^{-x} D^n (x^n e^x) = \sum_{r=0}^n \left[\frac{n!}{(n-r)!} \right]^2 \frac{x^{n-r}}{r!}.$$

18. Reduce the equation $4x^2 + 4xy + y^2 + 4x + y - 15 = 0$ to its canonical form and determine the nature of the conic.

19. If the astroid $x^{2/3} + y^{2/3} = c^{2/3}$ is the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$, prove that the parameters a and b are connected by the relation $a^2 + b^2 = c^2$.

$$20. \text{ Solve } \left(1 - y^2 + \frac{y^4}{x^2} \right) p^2 - 2 \frac{y}{x} p + \frac{y^2}{x^2} = 0.$$

21. Determine the equation of the cubic which has the same asymptotes as the curve $x^3 - 6x^2y + 11xy^2 - 6y^3 + 5x - 3y + 3 = 0$ and which passes through the points (0, 0), (1, 2), (2, 1).

P.T.O.

Group - C

Answer any *two* questions : $10 \times 2 = 20$

22. (a) Show that if y_1 and y_2 be solutions of equation

$$\frac{dy}{dx} + P(x)y = Q(x) \text{ and } y_2 = zy_1, \text{ then prove that}$$

$$z = 1 + ae^{-\int \frac{Q(x)}{y_1} dx}, \text{ } a \text{ is an arbitrary constant.}$$

- (b) Show that the points of inflection of the curve

$$y^2 = (x-a)^2(x-b) \text{ lie on the line } 3x + a = 4b.$$

5+5

23. (a) Prove that

$$(m+1)J_{m,n} = -\cos^m x \cos nx + mJ_{m-1,n-1} \text{ when}$$

$$J_{m,n} = \int \sin nx \cos^m x dx.$$

- (b) A circle of a given diameter d passes through the focus of a given conic and cuts it in four points whose distances from the focus are r_1, r_2, r_3, r_4 . Prove that

$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} = \frac{2}{l} \text{ and } r_1 r_2 r_3 r_4 = \frac{d^2 l^2}{e^2}. \quad 4+6$$

24. (a) If a_1, a_2, \dots, a_n be all positive real numbers, find the value of

$$\lim_{x \rightarrow \infty} \left\{ x - \sqrt[n]{(x-a_1)(x-a_2)\dots(x-a_n)} \right\}.$$

- (b) Find the ranges of values of x in which the curve

$$y = 3x^5 - 40x^3 + 3x - 20 \text{ is concave or convex.}$$

Also find the points of inflexion. 5+5

25. (a) Prove that the equations

$$2x = ae^{2\phi}, 2y = be^{\phi} \cosh \theta, z = ce^{\phi} \sinh \theta$$

determine a hyperbolic paraboloid and that $\theta + \phi$ is constant for a generator of one system and $\theta - \phi$ is constant for a generator of the other system.

- (b) Find the equation of the cylinder with guiding curve

$$2y^2 + 3z^2 = 1, x = 0 \text{ and generators parallel to}$$

$$x = y = z.$$

7+3