



বিদ্যাসাগর বিশ্ববিদ্যালয়

**VIDYASAGAR UNIVERSITY**

**Question Paper**

**B.Sc. General Examination 2023**

**(Under CBCS Pattern)**

**Semester — II**

**Subject : MATHEMATICS**

**Paper : DSC-1BT/2BT/3BT**

**( Differential Equations )**

**Full Marks : 60**

**Time : 3 hours**

*The figures in the right-hand margin indicate marks.*

*The symbols used have their usual meanings.*

Answer from **all** the Groups as directed.

**GROUP—A**

1. Answer **any ten** questions from the following :

2×10=20

- (a) Show that the following sets of functions are linearly independent.

$$e^{ax}, e^{bx}, e^{cx} \quad (a \neq b \neq c)$$

( 2 )

(b) Solve  $p \tan x + q \tan y = \tan z$ , where  $p = \frac{\partial z}{\partial x}$ ,

$$q = \frac{\partial z}{\partial y}.$$

(c) Solve  $y^2 + qx^2 = x^2 y^2 z^2$ , where  $p = \frac{\partial z}{\partial x}$ ,

$$q = \frac{\partial z}{\partial y}.$$

(d) Solve  $x dx + y dy + \frac{x dx + y dy}{x^2 + y^2} = 0$

(e) Solve  $(x^2 + y^2 + 2x) dx + 2y dy = 0$

(f) Solve  $\frac{dy}{dx} + \frac{\sin 2y}{x} = x^3 \cos^2 y$

(g) Solve  $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$

(h) Determine  $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$

(i) Reduce  $x^2 p^2 + y(2x+y)p + y^2 = 0$  to Clairaut's form by the substitution  $y = u$ ,  $xy = v$ .

(j) Solve  $py = p^2(x-b) + a$ , where  $p = \frac{dy}{dx}$ .

( 3 )

(k) Find the particular integral of the differential equation  $(D+2)y = e^{-2x} \sin 3x$ .

(l) Solve  $x^2 p + y^2 q = z^2$ , where  $p = \frac{\partial z}{\partial x}$ ,  $q = \frac{\partial z}{\partial y}$ .

(m) Classify the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + 3 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} - 2 \frac{\partial z}{\partial y} - 3 \frac{\partial z}{\partial x} - 4z = 0$$

(n) Find the orthogonal trajectories of the family of straight lines  $y = mx$ .

(o) Solve  $\frac{1}{(D-3)(D-2)} \log x$

### GROUP—B

2. Answer *any four* questions from the following :

5×4=20

(a) Solve by Lagrange's method of solution

$$x(y-z)p + y(z-x)q = z(x-y)$$

(b) Solve  $\frac{d^2 x}{dt^2} - 3x - 4y = 0$

$$\frac{d^2 y}{dt^2} + y + x = 0$$

(c) Solve

$$(xy \sin xy + \cos xy)y dx + (xy \sin xy - \cos xy)x dy = 0$$

( 4 )

(d) Solve  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3x^2e^{2x} \sin 2x$

(e) Solve  $\frac{dy}{dx} + \frac{y}{x} \log y = \frac{y}{x} (\log y)^2$

(f) Solve

$$(4x^3 - y^3 + 2xy^2)dx + (2x^2y - 3xy^2 + 4y^3)dy = 0$$

given  $y = 2$ , when  $x = 1$ .

### GROUP—C

3. Answer any **two** questions from the following :

10×2=20

(a) (i) Solve

$$x(y^2 + z)p - y(x^2 + z)q = z(x^2 + y^2) \quad 5$$

(ii) Find partial differential equation by eliminating the arbitrary function  $\phi$  from 5

$$\phi(x + y + z, x^2 + y^2 - z^2) = 0$$

(b) (i) Solve  $\cos x \frac{dy}{dx} - y \sin x = y^2$  5

(ii) Find the integral surface of

$$x^2p + y^2q + z^2 = 0$$

which passes through the hyperboloid

$$xy = x + y, z = 1. \quad 5$$

( 5 )

(c) (i) Solve  $x^3 \frac{dy}{dx} = y^3 + y^2 \sqrt{y^2 - x^2}$  5

(ii) Find an integrating factor of the equation  $(x^4 y^2 - y)dx + (x^2 y^4 - x)dy = 0$  and hence solve it. 5

(d) (i) Solve  $(y + z)p + (z + x)q = (x + y)$  by Lagrange's method. 5

(ii) Solve  $(p^2 + q^2)y = qz$  by Charpit's method. 5

★ ★ ★