



বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. General Examination 2023

(Under CBCS Pattern)

Semester — II

Subject : MATHEMATICS

Paper : DSC-1BT/2BT/3BT

(Differential Equations)

Full Marks : 60

Time : 3 hours

The figures in the right-hand margin indicate marks.

The symbols used have their usual meanings.

Answer from **all** the Groups as directed.

GROUP—A

1. Answer **any ten** questions from the following :
 $2 \times 10 = 20$

(a) Show that the following sets of functions are linearly independent.

$$e^{ax}, e^{bx}, e^{cx} \quad (a \neq b \neq c)$$

(2)

(b) Solve $p \tan x + q \tan y = \tan z$, where $p = \frac{\partial z}{\partial x}$,

$$q = \frac{\partial z}{\partial y}.$$

(c) Solve $y^2 + qx^2 = x^2y^2z^2$, where $p = \frac{\partial z}{\partial x}$,

$$q = \frac{\partial z}{\partial y}.$$

(d) Solve $xdx + ydy + \frac{xdx + ydy}{x^2 + y^2} = 0$

(e) Solve $(x^2 + y^2 + 2x)dx + 2ydy = 0$

(f) Solve $\frac{dy}{dx} + \frac{\sin 2y}{x} = x^3 \cos^2 y$

(g) Solve $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$

(h) Determine $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$

(i) Reduce $x^2 p^2 + y(2x+y) p + y^2 = 0$ to
Clairaut's form by the substitution $y = u$,
 $xy = v$.

(j) Solve $py = p^2(x-b) + a$, where $p = \frac{dy}{dx}$.

(3)

(k) Find the particular integral of the differential equation $(D + 2)y = e^{-2x} \sin 3x$.

(l) Solve $x^2 p + y^2 q = z^2$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$.

(m) Classify the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + 3 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} - 2 \frac{\partial z}{\partial y} - 3 \frac{\partial z}{\partial x} - 4z = 0$$

(n) Find the orthogonal trajectories of the family of straight lines $y = mx$.

(o) Solve $\frac{1}{(D-3)(D-2)} \log x$

GROUP—B

2. Answer **any four** questions from the following :
 $5 \times 4 = 20$

(a) Solve by Lagrange's method of solution

$$x(y-z)p + y(z-x)q = z(x-y)$$

(b) Solve $\frac{d^2 x}{dt^2} - 3x - 4y = 0$

$$\frac{d^2 y}{dt^2} + y + x = 0$$

(c) Solve

$$(xy \sin xy + \cos xy) y dx + (xy \sin xy - \cos xy) x dy = 0$$

(4)

(d) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3x^2e^{2x} \sin 2x$

(e) Solve $\frac{dy}{dx} + \frac{y}{x} \log y = \frac{y}{x} (\log y)^2$

(f) Solve

$$(4x^3 - y^3 + 2xy^2)dx + (2x^2y - 3xy^2 + 4y^3)dy = 0$$

given $y = 2$, when $x = 1$.

GROUP—C

3. Answer **any two** questions from the following :

10×2=20

(a) (i) Solve

$$x(y^2 + z)p - y(x^2 + z)q = z(x^2 + y^2) \quad 5$$

(ii) Find partial differential equation by
eliminating the arbitrary function ϕ
from 5

$$\phi(x + y + z, x^2 + y^2 - z^2) = 0$$

(b) (i) Solve $\cos x \frac{dy}{dx} - y \sin x = y^2$ 5

(ii) Find the integral surface of

$$x^2p + y^2q + z^2 = 0$$

which passes through the hyperboloid
 $xy = x + y, z = 1.$ 5

(5)

(c) (i) Solve $x^3 \frac{dy}{dx} = y^3 + y^2 \sqrt{(y^2 - x^2)}$ 5

(ii) Find an integrating factor of the equation $(x^4 y^2 - y)dx + (x^2 y^4 - x)dx = 0$ and hence solve it. 5

(d) (i) Solve $(y + z)p + (z + x)q = (x + y)$ by Lagrange's method. 5

(ii) Solve $(p^2 + q^2)y = qz$ by Charpit's method. 5

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