B.Sc./4th Sem (H)/PHYS/23(CBCS)

2023

4th Semester Examination

PHYSICS (Honours)

Paper : C 8-T

[Mathematical Physics - III]

[CBCS]

Full Marks : 40 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answer wherever necessary.
Symbols have their usual meaning.

1. Answer any ***five*** from the following : 2×5=10

   (i) Find the square root of the following $3 + 4i$.

   (ii) Prove that $(AB)^{-1} = B^{-1}A^{-1}$.

   (iii) In which domain(s) of the complex plane $f(z) = |x| - i|y|$ is an analytic function?

   (iv) Identify the zeroes, poles and essential singularities of $e^{\frac{1}{z}}$.

P.T.O.

(v) Simplify the expression $z = i^{-2i}$.

(vi) What is the Fourier transform of Dirac delta $\delta(x - x_0)$?

(vii) Write convolution theorem involving Fourier transform.

(viii) Prove eigenvalues of a hermitian matrix are real.

2. Answer any ***four*** from the following : 5×4=20

(i) Prove that $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta} = \frac{2\pi}{\sqrt{3}}$.

(ii) Find the Fourier transform of the normalised Gaussian distribution

$$f(t) = \frac{1}{\tau\sqrt{2\pi}} e^{-\frac{t^2}{2\tau^2}} \qquad -\infty \le t \le \infty$$

(iii) Find a function $f(z)$, analytic in a suitable part of the Argand diagram, for which

$$\operatorname{Re} f = \frac{\sin x}{\cosh 2y - \cos 2x}$$

Where are the singularities of $f(z)$?

(iv) Determine the types of singularities (if any) possessed by the following functions at $z = 0$ and $z = \infty$:

(a) $(z-2)^{-1}$,

(b) $(1+z^3)/z^2$,

(c) $\sinh(1/z)$,

(d) e^z/z^3.

(v) What is Cauchy Riemann condition? Apply on the function $f(z)=|z|^2$ and comment on its analyticity.

(vi) Using Cayley Hamilton Theorem find the inverse matrix

$$\begin{vmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{vmatrix}.$$

3. Answer any ***one*** from the following : 10×1=10

(i) (a) Find the Fourier transform of the given function

$$f(x) = \begin{array}{l} 1 \text{ for } |x| < a \\ 0 \text{ for } |x| > a. \end{array}$$

(b) Using contour integration to evaluate the real integral

$$\int_0^\infty \frac{1}{1+x^2}\,dx = \frac{\pi}{2}.$$

P.T.O.

(ii) (a)

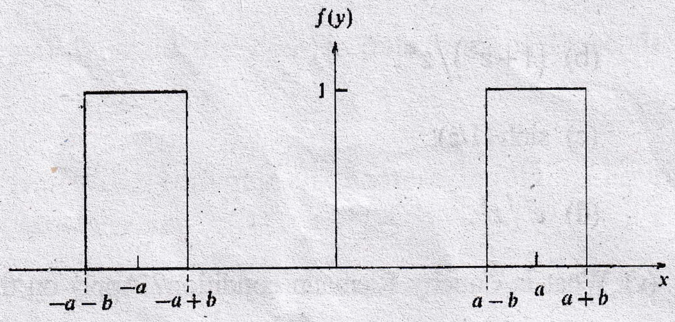

Fig.1

Find the Fourier transform of the function in figure 1 representing two wide slits by considering the Fourier transforms of

(A) two δ-functions, at $x = \pm a$,

(B) a rectangular function of height 1 and width $2b$ centred on $x = 0$. 2+2=4

(b) Find the Fourier transform of the function $f(t) = \exp(-|t|)$.

(A) By applying Fourier's inversion theorem prove that

$$\frac{\pi}{2}\exp(-|t|) = \int_0^\infty \frac{\cos \omega t}{1+\omega^2}.$$

(B) By making the substitution $\omega = \tan\theta$, demonstrate the validity of Parseval's theorem for this function. 3+3=6