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2022

3rd Semester Examination

MATHEMATICS (General)

Paper : DSC 1C/2C/3C-T

(Real Analysis)

[CBCS]

Full Marks : 60 Time : Three Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Group - A

Answer any *ten* of the following questions : 2×10=20

1. Prove that the set $A=\left\{-1,1-\frac{1}{2},\frac{1}{2},-\frac{1}{3},\frac{1}{3},\ldots\right\}$ is neither open nor closed.

2. If $x_n=\frac{1}{n}\sin\frac{n\pi}{2}$, show that the sequence $\{x_n\}$ converges.

3. Find the least upper bound of the set $\left\{\frac{(n+1)^2}{2n}, n\in N\right\}$.

4. Show that the series $\sum_{n=0}^{\infty}\frac{(-1)^n}{\log n}$ is conditionally convergent.

P.T.O.

5. Give an example of the open cover of the set (0, 1] which does not have a finite sub cover.

6. Examine the convergence of the series

$$\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1}-n\right).$$

7. Test the convergence of the given sequence of functions $\{f_n(x)\}$, where $\{f_n(x)\}=\frac{kx^2}{n};0\le x<k.$

8. Find the supremum and infimum, if exist, of

$$\left\{\frac{3n+2}{2n+1}:n\in\mathbb{N}\right\}.$$

9. If $\sum u_n^2$ and $\sum v_n^2$ are both convergent series, prove that the series $\sum u_n v_n$ is also convergent.

10. Prove that every infinite subset has a countable subset.

11. Show that the series $\sum\frac{\sin nx}{n^p}$ is uniformly convergent for all values of x and $p>1$.

12. Show that the sequence $\left\{nxe^{-nx^2}\right\}\forall n\in\mathbb{N}$ is not uniformly convergent on [0,1].

13. Prove that the set $\mathbb{N}\times\mathbb{N}$ is countable.

14. Prove that a monotonic sequence is never oscillatory.

15. Show that $\lim\limits_{n\to\infty}\left(\frac{n^n}{n!}\right)^{\frac{1}{n}}=e.$

Group - B

Answer any *four* of the following questions :

5×4=20

1. Show that the series $\sum_{n=0}^{\infty} \frac{x}{(nx+1)\{(n-1)x+1\}}$ is uniformly convergent on $[\delta, 1]$ for each $0<\delta<1$ but it is only point wise convergent on $[0, 1]$.

2. State and prove Cauchy's General Principle of convergence.

3. Define closed set. Prove that a set is closed iff its complement is open.

4. Examine the convergence of the series $\frac{1+a}{1+b}+\frac{(1+a)(2+a)}{(1+b)(2+b)}+\cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \infty (a,b \geq 0)$.

5. Prove that if a series $\sum u_n$ is convergent, then $\lim_{n\to\infty} u_n = 0$. Is the converse true? Justify with an example.

6. Determine the interval of convergence of the power series $\sum \frac{(-1)^{n+1}}{n}(x-1)^n$.

Group - C

Answer any *two* of the following questions :

10×2=20

1. (a) Show that every non-empty subset S of R which has an upper bound has the supremum.

P.T.O.

(b) Find the Supremum and infimum (if any) of the set, $X \subseteq R$; where $X = \left\{\frac{1}{n}, n \in N\right\}$.

(c) Find the derived set of the set $\left\{\frac{1}{m}+\frac{1}{n}+\frac{1}{p}; m, n, p \in N\right\}$. 4+4+2

2. (a) Define Dominated series. Show that the series $\sum_{n=0}^{\infty} \frac{\sin nx}{n^2}$ is dominated.

(b) Prove that a Dominated series (on an interval I) is uniformly convergent on I. 5+5

3. (a) Prove that the set $S = \left\{x : x \in \mathbb{Q}^+ \text{ and } 0 < x^2 < 3\right\}$ do not have any L.U.B in $\mathbb{Q}$.

(b) Prove that the series $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$. 5+5

4. (a) Show that if $f_n(x) = \frac{n^2 x}{1+n^4 x^2}$, then $\{f_n\}$ converges non-uniformly on [0.1].

(b) Prove that if the power series $\sum a_n x^n$ is such that $a_n \neq o \forall n \in \mathbb{N}$ and $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{R}$ then $\sum a_n x^n$ is convergent for $|x| < R$ and divergent for $|x| > R$.

5+5
