

2022

3rd Semester Examination

PHYSICS (Honours)

Paper : C 5-T

[Mathematical Physics - II]

[CBCS]

Full Marks : 40

Time : Two Hours

The figures in the margin indicate full marks.

*Candidates are required to give their answers
in their own words as far as practicable.*

Group - A

1. Answer any *five* of the following : 2×5=10

- (a) Define regular and apparent singular point.
- (b) What are Dirichlet conditions for a function to be piece-wise regular in a given interval ?
- (c) Write down the generating relation of Bessel function and show that $J_{n-1}(x) - J_{n+1}(x) = 2J'_n(x)$.
- (d) Define cyclic coordinates. Show that the generalized momentum conjugate to cyclic coordinate is conserved.

P.T.O.

(2)

- (e) Write down the Hermite's polynomial and hence show that $H'_n(x) = 2nH_{n-1}(x)$.
- (f) Write down the Laplace's equation in spherical polar coordinates.
- (g) Prove that $\Gamma(n+1) = n\Gamma(n)$, $n > 0$.
- (h) Derive the canonical equations of Hamiltonian.

Group - B

2. Answer any *four* of the following : 5×4=20

- (a) Express $f(x) = x^2$ as a Fourier's series in the interval $-\pi \leq x \leq \pi$, hence show that at

$$x = \pi, \quad \sum_{n=0}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}. \quad 4+1$$

- (b) Prove that the Legendre's polynomials

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n, \text{ symbols have their usual meaning.} \quad 5$$

- (c) Write down the Euler's equation in variational problems and using this show that the path of shortest (brachistos) time (chronos) of a particle is a *cycloid*. 1+4

(3)

- (d) Write down the Laplace's equation on a plane in terms of the polar coordinates. Solve it by the method of separation of variables and write the general expression of the solution which is finite at $r = 0$ and single valued in θ . 1+4
- (e) A simple pendulum consists of mass m_2 , with a mass m_1 at the point of support which can move on a horizontal line in the plane in which m_2 moves. Find the Lagrangian of the system and Lagrange's equations. 3+2
- (f) A bar of length L whose entire surface is insulated including its ends at $x = 0$ and $x = L$ has initial temperature $f(x)$. Determine the subsequent temperature of the bar. 5

Group - C

3. Answer any **one** of the following : 10×1=10

- (a) (i) State Hamilton's principle and derive Lagrange's equation of motion from it. Discuss how the result will be modified for non-conservative forces.
- (ii) From the generating function of Legendre's polynomials, show that
$$nP_n(x) = xP'_n(x) - P'_{n-1}(x).$$
- (iii) State the Parseval's identity of Fourier series. (1+4+1)+2+2

P.T.O.

(4)

(b) (i) Expand $f(x) = \sin x$, $0 < x < \pi$, in a Fourier cosine series.

(ii) Evaluate the value of integration

$$\int_0^1 x^4 (1-x)^3 dx, \text{ using Beta function.}$$

(iii) Write down the two dimensional wave equation in polar coordinates and solve it to find the eigen values and eigen functions in case of a circular membrane.

4+2+4